



On the Correlation Between Maximum Amplitude and Smoothed Monthly Mean Sunspot Number During the Rise of the Cycle (From $t=0$ –48 Months Past Sunspot Minimum)

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TECHNICAL PUBLICATION

ON THE CORRELATION BETWEEN MAXIMUM AMPLITUDE AND SMOOTHED MONTHLY MEAN SUNSPOT NUMBER DURING THE RISE OF THE CYCLE (FROM $t = 0$ –48 MONTHS PAST SUNSPOT MINIMUM)

1. INTRODUCTION

In the past, conventional onset for the start of a sunspot cycle has corresponded to the time when smoothed monthly mean sunspot number (i.e., the 12-mo moving average^{1, 2}) is at a minimum in value (minimum amplitude). Following this occurrence, smoothed monthly mean sunspot number usually increases in value in a straightforward way over a period of about 3 to 5 yr (on the basis of the “modern era” sunspot cycles 10–22) until it reaches a maximum in value (maximum amplitude), its occurrence denoting the conventional peak of the cycle. In actuality, solar minimum should be regarded as a 2–3 yr interval when solar activity is relatively low^{3, 4} and, likewise, solar maximum as a 3–4 yr interval when solar activity is relatively high.^{5, 6} The conventional onsets and peaks, then, occur sometime during these extended periods of time.

It is now apparent⁷ that 1996 marks the minimum occurrence year for cycle 23 (based on annual averages of sunspot number) and that cycle 23 is in its rising phase, racing towards maximum, probably in either 1999 or 2000. While true, placement⁸ of its conventional onset (based on smoothed monthly mean sunspot number) has been difficult to assign, owing to two rather unusual occurrences: First, cycle 23 is the first cycle to have had its first occurrence of a high-latitude spot group (≥ 25 deg) *in conjunction* with a minimum in smoothed monthly mean sunspot number (May 1996). All previous cycles have had their first occurrence of a high-latitude spot group to precede minimum amplitude by at least 3 mo.^{3, 8} Second, following minimum amplitude, smoothed monthly mean sunspot number almost always rises unabatedly towards maximum amplitude (having positive first differences), being associated with the transition from old to new cycle spots and the strengthening of the cycle with longer elapsed time from minimum. For the modern era cycles 10 and 21, a slight dip is noted to have occurred in the vicinity of their respective minimum amplitudes (occurring a few to several months after their conventional minimums), where each dip measured about 10–20 percent larger than their respective conventional minimum amplitudes. For cycle 23, a secondary dip, measuring only 4 percent larger (8.3 versus 8.0) occurred almost immediately (in August 1996) following its apparent conventional minimum (May 1996); this secondary dip being closely associated with the months of lowest monthly mean sunspot number (September and October 1996), which also were the months of greatest number of spotless days, and with the minimum in the 12-mo moving average in number of spot groups (August 1996). Following this secondary dip, smoothed monthly mean sunspot number (and number of groups) steadily rose, with new cycle spots finally becoming the dominant contributor to sunspot area about April 1997.

Recently, Wilson et al.⁹ identified several statistically important associations that relate to the size or maximum amplitude of the sunspot cycle. Of particular interest here is the “maximum-minimum effect,” an inferred association between the size of the cycle at maximum amplitude and the size of the cycle at onset (i.e., minimum amplitude). Wilson et al. found that, on the basis of this maximum-minimum effect, one could use the size of the cycle at onset to estimate the size of the cycle at maximum, typically, to within ± 30 percent, whether one used all cycles 1–22 or just the modern era sunspot cycles 10–22, so named because of the completeness of the sunspot record (i.e., they are the most reliably known). It was further noted that cycle 19 (the largest cycle on record) deviated the most with respect to this inferred association. Excluding cycle 19 caused the coefficient of correlation r to increase from 0.56 to 0.72 for the overall data set (cycles 1–22) and the standard error of estimate se to decrease from about 35 to 26.

In two other papers, Wilson et al.^{8, 10} demonstrated that the existence of the maximum-minimum effect, when combined with the “Waldmeier effect,” an inverse relationship between the size of the cycle at maximum amplitude and the ascent duration,⁹ suggests a means whereby one can determine rather quickly the apparent rise (i.e., *fast* versus *slow* riser) and amplitude (i.e., *large* versus *small* maximum amplitude) classes of an unfolding sunspot cycle. Cycles that have smoothed monthly mean sunspot number values equal to or above the mean curve for cycles 1–22 during the early rise phase of the cycle nearly always (10 out of 11) turn out to be fast risers (ascent duration < 48 mo) of larger than average maximum amplitude ($RM \geq 113.2$), while cycles that have smoothed monthly mean sunspot number values below the mean cycle curve during the early rise phase of the cycle nearly always (10 out of 11) turn out to be slow risers (ascent duration ≥ 48 mo) of smaller than average maximum amplitude ($RM < 113.2$). Furthermore, it was found that the probable rise and amplitude classes of the unfolding sunspot cycle can usually be determined within the first 12–16 mo of the cycle. While quite useful, the technique did not yield a specific estimate of the actual size for the unfolding cycle (i.e., its maximum amplitude), except by comparison to the means of fast- and slow-rising cycles.⁸ On the basis of modern era cycles, fast risers usually have a maximum amplitude of about 144 ± 29 , while slow risers usually have a maximum amplitude of about 84 ± 17 (i.e., the bulk of the cycles have observed maximum amplitudes that lie within 20 percent of the means). Consequently, providing that cycle 23 is a *well-behaved* fast or slow riser, its RM is expected to be either < 173 (fast riser) or < 101 (slow riser).

In this paper, on the basis of the modern era sunspot cycles, we examine more closely the predictive aspects of the current value of smoothed monthly mean sunspot number (i.e., the most recently available value) with regards to the size of the later occurring maximum amplitude, from cycle onset to 48 mo into the cycle. We show that the current value of smoothed monthly mean sunspot number during the rise of the cycle, indeed, can be used with increasing confidence and accuracy to estimate the size of maximum amplitude for an unfolding sunspot cycle, especially after the first 2 yr of cycle rise, and we apply this technique to cycle 23.

2. THE MAXIMUM-MINIMUM EFFECT

Figure 1 displays the scatter plot of maximum amplitude R_M versus minimum amplitude R_m , separately, for the two cases of all cycles 1–22 (left) and modern era cycles 10–22 (right). In both plots, the inferred linear regression (denoted as \hat{y}) is depicted as the thick diagonal line running from lower left to upper right. Likewise, in both plots, the inferred linear regression (denoted as y') is depicted as the thin diagonal line running from lower left to upper right for the case when cycle 19 is ignored. Ignoring cycle 19 changes the coefficient of correlation r from 0.56 to 0.72 for cycles 1–22 and from 0.47 to 0.74 for cycles 10–22; the coefficient of determination r^2 , a measure of the amount of variance that the inferred regression can explain, from 0.31 to 0.52 for cycles 1–22 and from 0.22 to 0.54 for cycles 10–22; and the standard error of estimate se from about 35 to 26 for cycles 1–22 and from about 38 to 24 for cycles 10–22. Furthermore, ignoring cycle 19 lowered the y -axis intercept (the first constant in the regression equation) from about 78.0 to 67.6 for cycles 1–22 and from about 89.8 to 73.3 for cycles 10–22, and it raised the inferred slope (the second constant in the regression equation) from about 6.0 to 6.9 for cycles 1–22 and from about 5.2 to 6.7 for cycles 10–22.

Above each scatter plot in figure 1 is a display of the ratio of observed to predicted maximum amplitude R_M (from the regression fits), indicating how well the inferred linear regressions work at predicting the size of the cycle at cycle onset (for the cases that include cycle 19). For the bulk of the samples, the observed value of maximum amplitude is found to lie within the range of ± 30 percent of the predicted value of maximum amplitude. For the modern era cycles, we find that 10 out of the 13 cycles had a later occurring maximum amplitude that lie within the ± 30 percent range of the predicted maximum amplitude, and although not shown, we note here that, had we chosen to ignore cycle 19, the ratio would have been 12 out of 12 cycles having their later occurring observed maximum amplitudes within the ± 30 percent range bounding their predicted maximum amplitudes. (In figure 1, SCN refers to the sunspot cycle number.)

Because fast risers grow in sunspot number more rapidly than slow risers, there is a natural migration of fast risers (cycles of shorter than average ascent duration that almost always are larger than average size) to the right and of slow risers (cycles of average to longer than average ascent duration that almost always are smaller than average size) to the left in scatter plots of R_M versus $R(t)$, where t refers to the elapsed time from sunspot minimum. Thus, as the elapsed time from sunspot minimum becomes longer, the correlation of R_M versus $R(t)$ should strengthen and the standard error of estimate, likewise, should become smaller, indicating that we should be able to more accurately predict the size of the later occurring maximum amplitude (as the cycle gets progressively closer and closer to maximum amplitude occurrence). In the following section, we will examine these particular issues, especially as they apply to cycle 23.

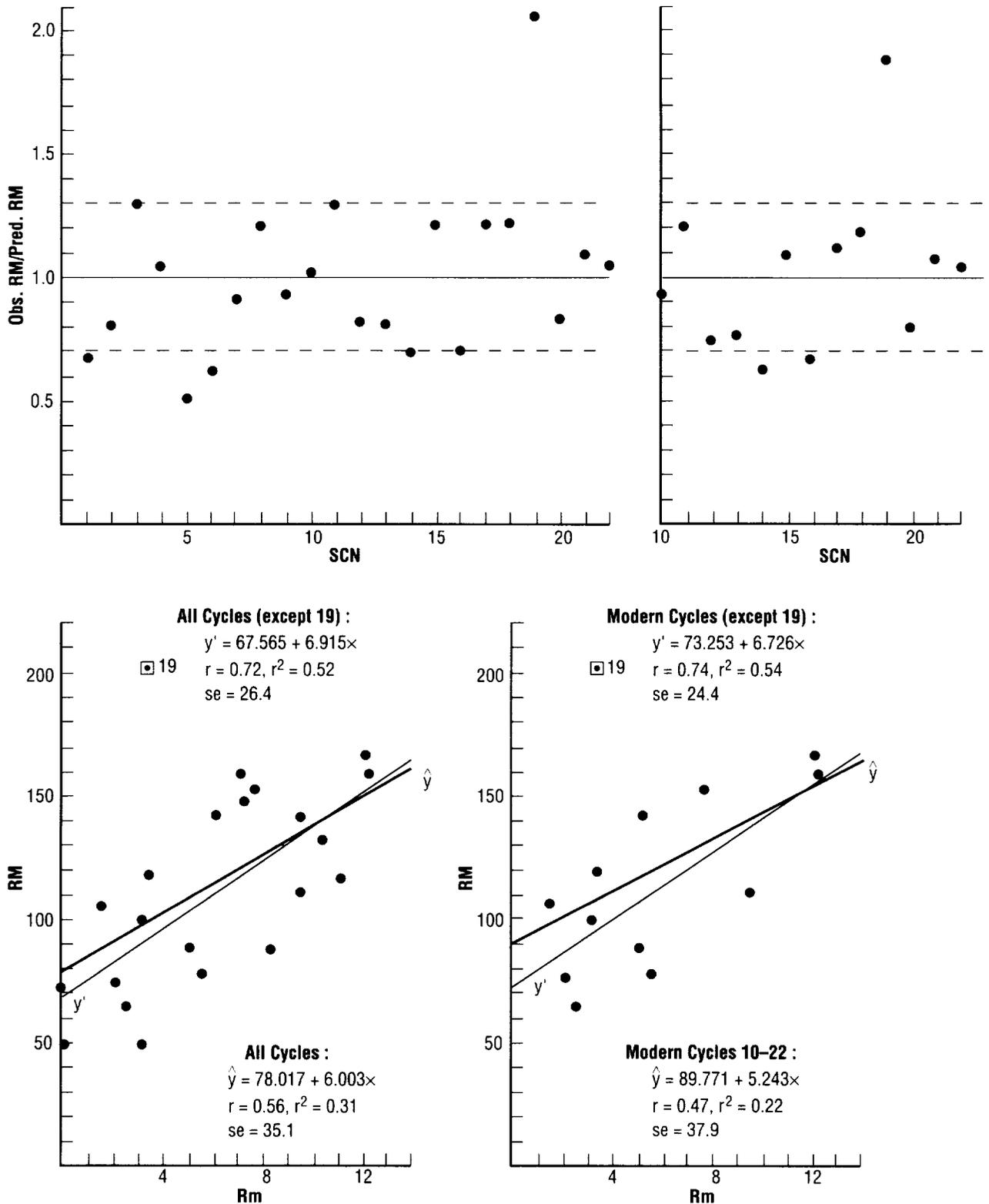


Figure 1. The “maximum-minimum effect” for cycles 1–22 (left) and cycles 10–22 (right). The ratio of observed to predicted RM (maximum amplitude) based upon the inferred regressions (top).

3. RESULTS AND DISCUSSION

Figure 2 displays the behavior of the coefficient of correlation r , the standard error of estimate se , the y -axis intercept a , and the slope b of the inferred regressions during the interval of elapsed time from sunspot minimum (i.e., $t = 0$) through 48 mo. Identified across the bottom are the occurrences of maximum amplitude for cycles 10–22 (except cycles 12 and 16 which occurred later than 50 mo past minimum, respectively, at 60 and 56 mo past minimum). Identified across the top are the levels of confidence for the inferred regressions, where ≥ 90 percent indicates a marginally significant result, ≥ 95 percent a statistically significant result, and so forth. Figure 2 suggests that for the modern era cycles (including cycle 19) the inferred regressions grow stronger as the cycle progresses from minimum to maximum. From 11 mo past minimum, the regressions are considered statistically important, and from 15 mo past minimum they are considered very important. The inferred slope changes only slightly beyond $t = 24$ mo, and r is ≥ 0.9 for $t \geq 30$ mo. The standard error dips below 25 units of smoothed monthly mean sunspot number for $t \geq 24$ mo and below 20 units for $t \geq 28$ mo past minimum. Table 1 lists the actual computed values for r , se , a , and b that are displayed pictorially in figure 2, and it gives the value for the t -statistic regarding the statistical significance of the inferred regressions, where a sample size of 13 cycles is noted to have 11 degrees of freedom and results in the ± 90 -percent level of confidence being $t = 1.796$, the ± 95 -percent level of confidence level being $t = 2.201$, and the ± 99 -percent level of confidence being $t = 3.106$.

Figure 3 depicts the scatter plots of maximum amplitude RM versus $R(t)$ for elapsed time past sunspot minimum $t = 12, 18, 24,$ and 30 mo. Clearly, as the cycle progresses towards maximum amplitude, the inferred correlation between RM and $R(t)$ strengthens and the accuracy of the predicted RM is much improved, becoming quite good for $t \geq 24$ mo. While true, it should be noted that even early on in the cycle, when the $R(t)$ value exceeds the median value (the thin vertical line), this can be taken as strongly indicating that the cycle will have a maximum amplitude larger than average in size, and that when $R(t)$ is below the median, this can be taken as strongly indicating that the cycle will have a maximum amplitude smaller than average in size.

Figure 4 compares the ratios of observed to predicted RM for the regressions that are shown in figure 3. For $t = 12$ mo, 9 of 13 cycles are found to have their later occurring observed RM within the range of ± 30 percent of their predicted values. For $t = 18$ mo, 11 of 13 cycles meet this condition. For $t \geq 24$ mo, all cycles (13 of 13 cycles) are found to have their observed RM within the range of ± 30 percent of their predicted values, and to within ± 21 percent for $t \geq 30$ mo.

Table 2 shows the number of cycles out of a sample of 13 cycles (i.e., the modern era cycles) that had an observed to predicted RM ratio within the various stated bounds for $t = 0$ to 48 mo. It is apparent that from about $t = 20$ mo, the observed RM usually (i.e., 10 or more out of 13 cycles) is found to lie within the range of ± 25 percent (or better) of the predicted value.

Levels of Confidence For The Inferred Regressions

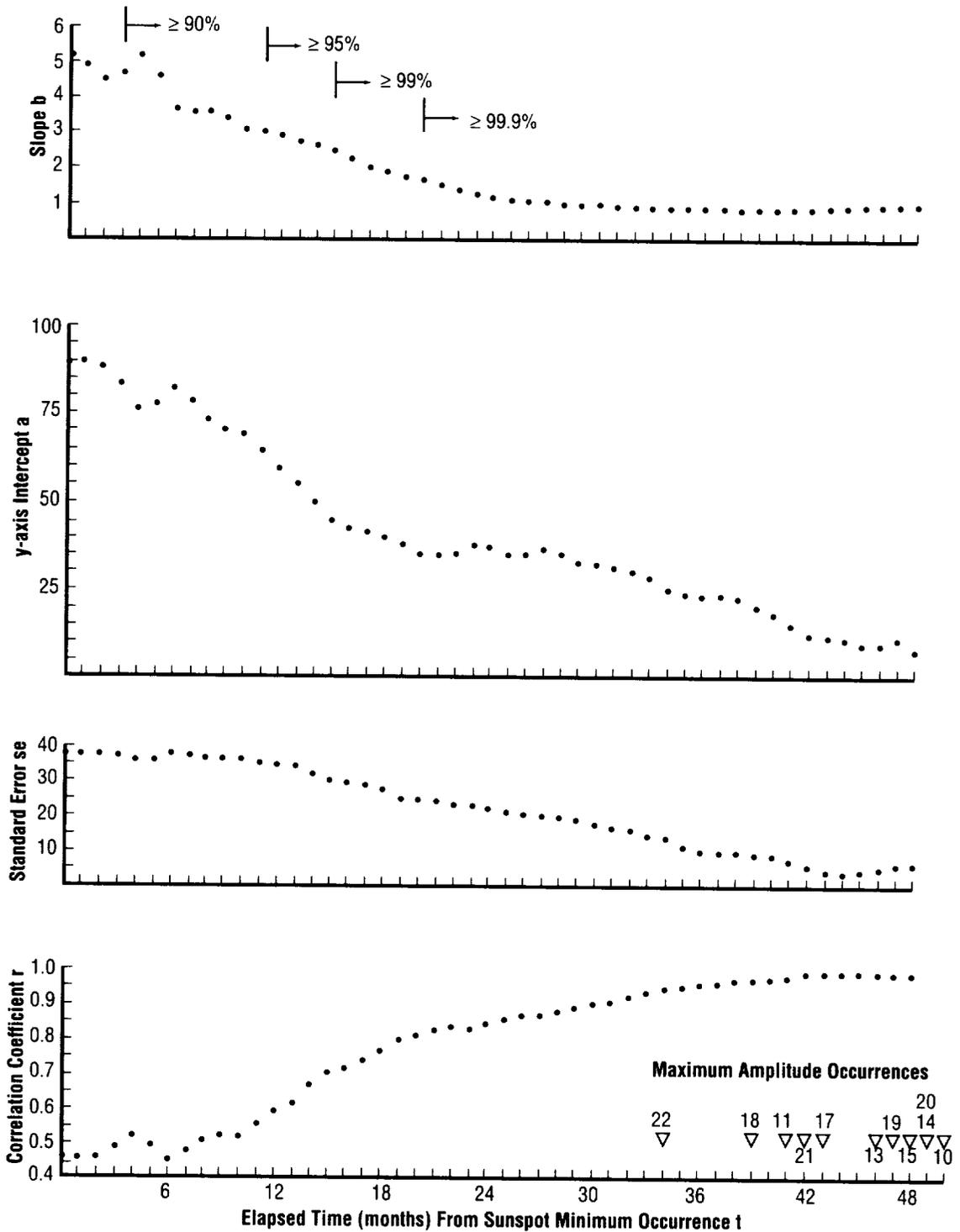


Figure 2. Behavior of r , se , a , and b for $t = 0$ to 48 mo. Correlations are marginally significant (≥ 90 percent) from $t = 3$ mo, statistically significant (≥ 95 percent) from $t = 11$ mo, and so forth. The placements of RM for the modern era sunspot cycles is shown across the bottom (except for cycles 12 and 16, which had ascent durations > 50 mo).

Table 1. Summary of linear regression fits for maximum amplitude versus $R(t)$ values based on sunspot cycle numbers 10–22.

t	r	se	a	b	t-statistic
0	0.469	37.9	89.771	5.243	1.76
1	0.463	38.0	90.133	4.889	1.73
2	0.465	38.0	88.883	4.581	1.74
3	0.487	37.5	84.134	4.760	1.85
4	0.526	36.5	77.039	5.252	2.05
5	0.498	37.2	78.265	4.634	1.91
6	0.457	38.2	82.684	3.738	1.71
7	0.480	37.6	79.676	3.600	1.81
8	0.512	36.9	74.112	3.635	1.98
9	0.525	36.5	70.509	3.478	2.05
10	0.524	36.5	69.310	3.151	2.04
11	0.559	35.6	64.791	3.062	2.24
12	0.590	34.6	59.355	2.982	2.42
13	0.617	33.8	55.623	2.804	2.60
14	0.671	31.8	50.303	2.717	3.00
15	0.705	30.4	45.703	2.576	3.30
16	0.721	29.7	43.173	2.354	3.45
17	0.741	28.8	42.283	2.126	3.66
18	0.769	27.4	40.361	1.970	3.99
19	0.799	25.8	37.977	1.859	4.40
20	0.816	24.8	35.860	1.757	4.68
21	0.827	24.1	35.199	1.615	4.88
22	0.835	23.6	36.506	1.457	5.04
23	0.835	23.6	38.340	1.315	5.04
24	0.846	22.9	38.108	1.231	5.26
25	0.863	21.7	35.921	1.194	5.67
26	0.872	21.0	36.024	1.135	5.90
27	0.878	20.5	37.301	1.070	6.08
28	0.886	19.9	35.937	1.049	6.34
29	0.896	19.1	33.849	1.046	6.70
30	0.906	18.2	33.319	1.018	7.10
31	0.915	17.3	31.870	0.997	7.52
32	0.924	16.4	30.602	0.973	8.02
33	0.938	14.9	29.334	0.955	8.98
34	0.950	13.4	26.935	0.954	10.09
35	0.959	12.2	24.475	0.953	11.23
36	0.967	10.9	24.118	0.932	12.59
37	0.971	10.3	24.251	0.910	13.47
38	0.972	10.1	23.332	0.899	13.72
39	0.973	9.9	21.100	0.904	13.99
40	0.977	9.2	18.438	0.915	15.20
41	0.982	8.1	16.204	0.916	17.24
42	0.989	6.3	13.302	0.931	22.17
43	0.994	4.7	11.399	0.946	30.14
44	0.995	4.3	10.524	0.956	33.05
45	0.994	4.7	9.591	0.971	30.15
46	0.991	5.7	9.704	0.975	24.57
47	0.989	6.3	10.150	0.974	22.17
48	0.988	6.6	7.871	1.000	21.21

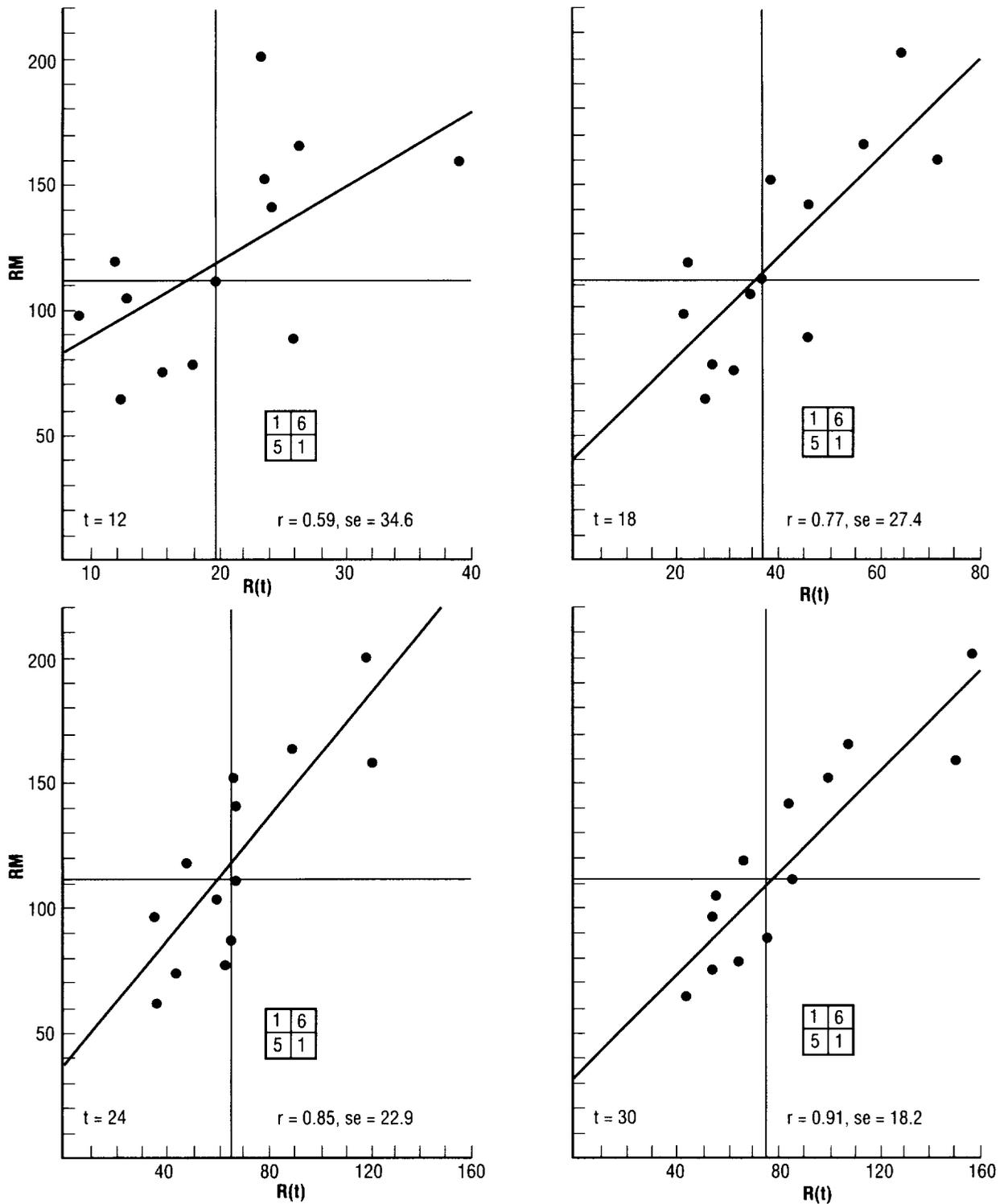


Figure 3. Scatter plots of RM versus $R(t)$ for $t = 12, 18, 24,$ and 30 mo. The diagonal lines are the inferred linear regressions, and the thin vertical and horizontal lines are the median values for the parameters. The small four-squared boxes show the distributions by quadrant for each correlation. On the basis of Fisher's exact test for 2×2 contingency tables, we compute the probability of obtaining the observed distribution, or one more suggestive of a departure from independence, to be 2.5 percent.

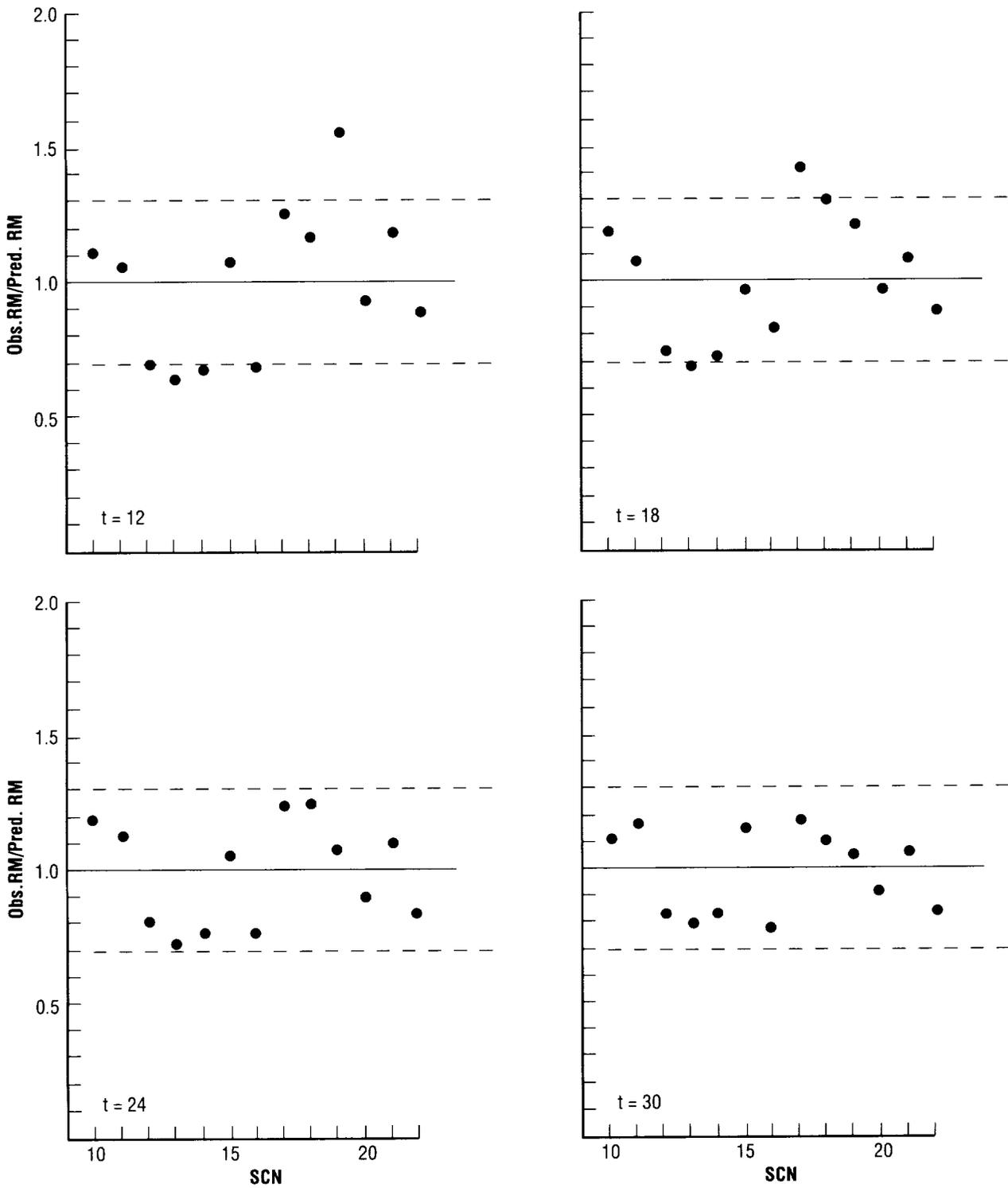


Figure 4. Ratio plots of observed to predicted RM for $t = 12, 18, 24,$ and 30 mo. SCN refers to the solar cycle number.

Table 2. Distribution of ratios as a function of t for sunspot cycle numbers 10–22 ($N=13$).

t	Ranges of Ratio (Obs. RM/Pred. RM)							
	0.50–1.50	0.60–1.40	0.70–1.30	0.75–1.25	0.80–1.20	0.85–1.15	0.90–1.10	0.95–1.05
0	12	12	10	9	7	5	4	1
1	12	12	10	8	6	5	4	1
2	12	12	10	8	6	5	5	1
3	12	12	10	8	6	5	5	2
4	12	12	10	10	6	5	3	1
5	12	12	10	8	7	4	3	1
6	12	12	10	8	7	5	3	3
7	12	12	10	8	7	5	3	3
8	12	12	10	8	7	5	4	1
9	12	12	8	8	7	5	4	0
10	12	12	8	8	7	5	5	0
11	12	12	9	8	7	6	4	0
12	12	12	9	7	7	5	4	0
13	13	12	9	7	7	5	3	1
14	13	13	9	7	7	4	3	3
15	13	12	10	8	6	5	3	1
16	13	12	9	8	6	5	4	3
17	13	12	10	7	7	5	4	3
18	13	12	11	8	8	5	4	2
19	13	13	11	9	8	6	3	2
20	13	13	11	10	8	6	3	2
21	13	13	12	10	8	6	2	2
22	13	13	13	10	8	5	2	1
23	13	13	13	10	8	5	2	1
24	13	13	13	11	8	6	3	0
25	13	13	13	13	8	5	4	0
26	13	13	13	13	7	5	4	1
27	13	13	13	13	7	6	4	1
28	13	13	13	13	9	6	4	1
29	13	13	13	13	9	8	3	1
30	13	13	13	13	12	7	3	1
31	13	13	13	13	12	8	3	1
32	13	13	13	12	12	8	5	1
33	13	13	13	12	12	10	6	2
34	13	13	12	12	12	12	6	3
35	13	13	12	12	12	12	9	5
36	13	13	12	12	12	12	10	6
37	13	13	13	12	12	12	10	7
38	13	13	12	12	12	12	11	8
39	13	13	12	12	12	12	11	8
40	13	13	13	12	12	12	11	9
41	13	13	13	12	12	12	11	10
42	13	13	13	13	13	12	11	9
43	13	13	13	13	13	13	13	10
44	13	13	13	13	13	13	13	12
45	13	13	13	13	13	13	12	10
46	13	13	13	13	13	13	12	9
47	13	13	13	13	13	12	12	8
48	13	13	13	13	13	12	12	7

Figure 5 displays estimates of cycle 23's RM using two different onsets: May 1996 (the lower, heavier line) and August 1996 (the higher, thinner line). Accepting May 1996 as the official onset for cycle 23 strongly suggests that its RM will be about the size of the mean cycle. Estimates for RM have become progressively smaller with time for $t = 0$ –17 mo, decreasing from about 131.7 ± 39.5 (at $t = 0$, with $R(0) = 8.0$) to 110.3 ± 33.1 (at $t = 17$, with $R(17) = 32.0$). On the other hand, accepting August 1996 as the official onset for cycle 23 strongly suggests that its RM will be larger than the size of the mean cycle. Estimates for RM have remained fairly stable (although there is a hint of an upward progression since about February 1997, $t = 6$ mo) with time for $t = 0$ –14 mo, being about 133.3 ± 40.0 (at $t = 0$, with $R(0) = 8.3$) to 137.2 ± 41.2 (at $t = 14$, with $R(14) = 32.0$). Thus, a *divergence* in prediction has become apparent, which strictly relates to the choice of onset date.

Previously, on the basis of various models and precursor techniques, it has been found that cycle 23 should be above average in size. For example, Kopecky¹¹ and Wilson,¹² on the basis of the “odd-even effect,” have suggested that cycle 23 should have a maximum amplitude that will be larger than average in size, probably of comparable or larger size as compared to that of cycle 22's (RM = 158.5). Similar findings have continued to be echoed.^{8, 13–20} In fact, a consensus prediction²¹ of about 160 ± 30 remains the best guess for the size of cycle 23's RM. Improvements in this estimate should be available later this year, using curve-fitting algorithms (such as the one described by Hathaway et al.²²).

Because the consensus is that cycle 23 will turn out to be a fast-rising, larger than average size cycle, in contrast to a slow-rising, smaller than average size cycle, this seems to suggest that the choice of May 1996 as the official onset for cycle 23 is *wrong* and will lead to the specious result that its RM will be only of about average size (i.e., the mean cycle) or, perhaps, even smaller. On the other hand, the choice of August 1996 as the official start for cycle 23 generates a prediction that is in much better agreement with the consensus prediction. Therefore, we suggest that for the purposes of solar activity prediction, the secondary minimum of August 1996 be used as the official (preliminary) start of cycle 23.

In conclusion, this study has demonstrated that the current value of smoothed monthly mean sunspot number during the rise from sunspot minimum to maximum can be used to provide a reasonably accurate estimation for the size of the later occurring maximum amplitude from a few to several years in advance of its occurrence, being particularly useful from about 18 mo past sunspot minimum. For cycle 23, the official start, while controversial, appears to yield estimates of RM closer to the consensus prediction when one accepts August 1996 as the official start for cycle 23, rather than May 1996. The current estimate for cycle 23's RM based on the described method and using August 1996 as the official start and the most recently available smoothed monthly mean sunspot number value of 32.0 (October 1997, $t = 14$ mo) as the independent variable is that it will be larger than average in size, measuring about 137.2 ± 41.2 . Because there is a hint of a rising estimate with the progression of time, it may be that our estimate will become slightly larger over the next several months. This may be an indication that the actual value of RM for cycle 23 will be in the upper portion of the prediction interval. (i.e., >137.2).

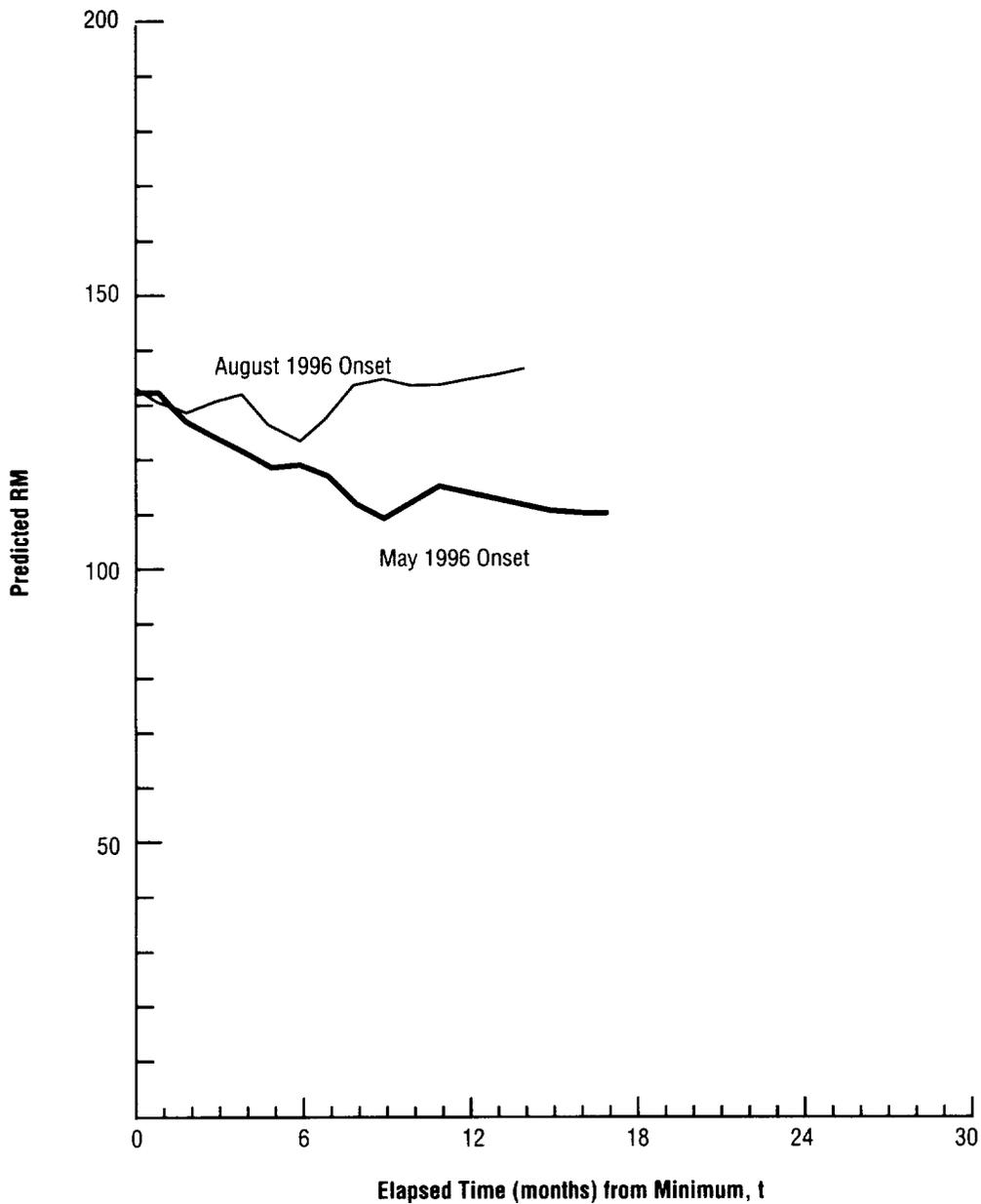


Figure 5. Predicted RM for cycle 23 using two different onset dates: May 1996 (heavier line) and August 1996 (thinner line). An onset date of August 1996 yields estimates for RM that are in better agreement with the consensus prediction. Please note that the actual prediction is ± 30 percent bounding the estimates.

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